Introduction to Linear Differential Equations

in the Complex Domain – about 20 lectures, 40 hours

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The aim of the course is to provide basic notions about linear systems of differential equations in the complex domain, monodromy data, isomonodromy deformations. These notions play an important role in modern mathematical physics, for example in integrable systems.

– Existence and uniqueness theorems in the complex domain.
– Linear systems
– Singularities and monodromy
– Classification of isolated singularities of linear systems (first and second kind).
– Linear systems with singularities of first kind (Fuchsian systems). Reduction to Birkhoff normal form.
– Linear equations of order $n$. Riemann and Gauss equations.
– Review of Poincaré asymptotics.
– Linear Systems with singularities of the second kind.
– Unramified singularities. Reduction to Birkhoff normal form. Invariants. Stokes phenomenon (some examples, such as the Bessel equation).
– Ramified singularities (some examples, such as the Airy equation).
– Global description. Monodromy data.
– Linear systems depending on parameters.

Prerequisites: Complex analysis, theory of analytic functions in one complex variable (see Reference 4. below).

Basic References


3. E.L. Ince: *Ordinary Differential Equations*

4. V.I. Smirnov: *A course of higher mathematics. Vol. 3. Part 2: complex variables, special functions*


9. Y. Sibuya: *Linear Differential Equations in Complex Domain; Problems of Analytic Continuation.* Translations of Mathematical Monographs 82, AMS.