Introduction to Linear Differential Equations

in the Complex Domain and Isomonodromic Deformations

about 30 hours

Davide Guzzetti, Fall 2020

The aim of the course is to provide basic notions about linear systems of differential equations in the complex domain, monodromy data, isomonodromy deformations. These notions play an important role in modern mathematical physics, for example in integrable systems.

– Existence and uniqueness theorems in the complex domain.

– Linear systems

– Singularities and monodromy

– Classification of isolated singularities of linear systems (first and second kind).

– Linear systems with singularities of first kind (Fuchsian systems). Reduction to Birkhoff normal form.

– Linear equations of order n. Riemann and Gauss equations.

– Review of Poincaré asymptotics.

– Linear Systems with singularities of the second kind.

– Unramified singularities. Reduction to Birkhoff normal form. Invariants. Stokes phenomenon (some examples, such as the Bessel equation).

– Global description. Monodromy data.

– Linear systems depending on parameters.

– Monodromy preserving deformations. Examples. Painlevé equations.

Prerequisites: Complex analysis, theory of analytic functions in one complex variable (see Reference 4. below).

Basic References

1. W. Wasow: *Asymptotic Expansions for Ordinary Differential Equations.*

2. E.A. Coddington, N. Levinson: *Theory of Ordinary Differential Equations.*

3. E.L. Ince: *Ordinary Differential Equations*

4. V.I. Smirnov: *A course of higher mathematics. Vol. 3. Part 2: complex variables, special functions*

5. K. Iwasaki, H. Kimura, S. Shimomura, M. Yoshida: *From Gauss to Painlevé.*

6. W. Balser, W.B. Jurkat, D.A. Lutz: *"Birkhoff Invariants and Stokes' Multipliers for Meromorphic Linear Differential Equations".* Journal of Mathematical Analysis and Applications **71**, 48-94 (1979).

7. W. Balser, W.B. Jurkat, D.A. Lutz: *A Genearal Theory of Invariants for Meromorphic Differential Equations.* Funkcialaj Evacioj, **22**, (1979). Part I (pages 197-221), Part II (pages 257-283)

8. A. Fokas, A.R. Its, A.A. Kapaev, V.Y. Novokshenov: *Painlevé Transcendents, The Riemann-Hilbert approach.*

9. Y. Sibuya: *Linear Differential Equations in Complex Domain; Problems of Analytic Continuation.* Translations of Mathematical Monographs 82, AMS.

10. D. Guzzetti: *Introduction to Linear Differential Equations in the Complex Domain, and Isomonodromy Deformations*, Lecture Notes for e Ph.D. course at SISSA.