# Overview

**Title:** Theory and Applications of Random Matrices
**Instructor:** Fabio Deelan Cunden (SISSA) fcunden@sissa.it
**When:** 20 hours, Feb-Mar 2020

This course is an introduction to the theory of random matrices, one of the most active research topics in contemporary mathematical physics and probability.

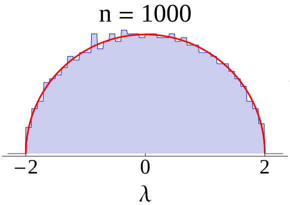
**Guiding question:**
can we describe the (statistical) behaviour of the eigenvalues of a random matrix?

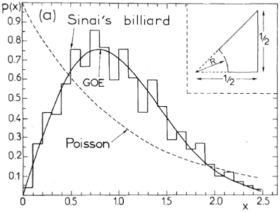
**Take-home message:**
"random matrices" are very *special*. In particular, the larger the size of the matrix the more predictable they are. For example, a $N \times N$ hermitian matrix with normally distributed entries has 2-norm very nearly $N$, and largest eigenvalue $2\sqrt{N}$. If you sort and plot the eigenvalues, you will get nearly the same picture each time.

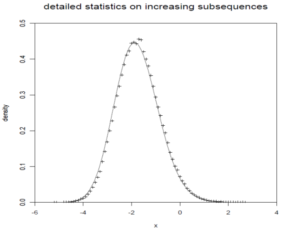
This is the beauty of a random matrix; it has "more structure" than a fixed matrix.

# Brief history and a few pictures

- 1897: Hurwitz (invariant theory)
- 1928: Wishart (multivariate statistics)
- 1955: Wigner (nuclear physics)
- 1960's: Mehta and Dyson (quantum mechanics)
- 1967: the first and influential book *Random matrices* by Mehta
- 1972: Montgomery and Dyson (random matrices and number theory)
- 1980's: quantum chaos, field theory, geometry and combinatorics
- since 1990's: random matrices are studied more, and more extensively in connection to other areas of mathematics.



# Universality classes

**Why?** One possible answer is...

...interest in random matrices has been spurred by the scientific hypothesis that large random matrices yield models for complex systems comprised of many highly correlated components. Such systems are ubiquitous in mathematics and nature (energy levels of heavy nuclei or chaotic quantum billiards, zeros of L-functions, random growth models, etc.) but are not within the purview of classical scalar probability theory, whose limit theorems usually apply to systems of weakly correlated components.

**Random matrix universality class(es):**

$$
\begin{array}{cccccc}
\text{Random matrix } A & \rightsquigarrow & \text{explicit formula } F_N(x) & \overset{N\to\infty}{\rightsquigarrow} & F_{RMT}(x) \\
\text{(Integrable) probability model} & \rightsquigarrow & \text{explicit formula } G_N(x) & \overset{N\to\infty}{\rightsquigarrow} & F_{RMT}(x) \\
\text{Probability model} & \rightsquigarrow & \text{no explicit formula ???} & \overset{N\to\infty}{\rightsquigarrow} & F_{RMT}(x)
\end{array}
$$

See abstract and a (tentative) list of topics online. Prerequisites: basic linear algebra and probability.

References:


- M. L. Mehta, Random Matrices, 1967.
- G. W. Anderson, A. Guionnet, O. Zeitouni, An introduction to Random Matrices, 2005.
- P. J. Forrester, Log-gases and Random Matrices, 2010.
- T. Tao, Topics in Random Matrix Theory, 2012.
- G. Livan, M. Novaes, P. Vivo, Introduction to Random Matrices - Theory and Practice, 2018.