

# Affine Lie algebras and Physics

Yegor Zenkevich, [yegor.zenkevich@gmail.com](mailto:yegor.zenkevich@gmail.com)

Fall 2020,  
Tuesdays, 14:00–16:00, Zoom

Affine Lie algebras are senior cousins of compact Lie algebras, all together belonging to the class of Kac-Moody algebras. The generators and relations of affine Lie algebras can be written using the Cartan matrix just as in the case of compact ones, the only difference being that the matrix is now positive *semi*-definite with a one-dimensional null eigenspace. This seemingly minor change, however, has nontrivial implications for the structure of the algebras: they become infinite-dimensional.

In this course we will use a hands-on approach to build and investigate affine Lie algebras and their representations. In the process we will

- classify all affine Lie algebras and draw the corresponding Dynkin diagrams
- learn about central extensions of loop algebras,
- study modifications of loop algebras arising from twisted boundary conditions along the loop,
- draw several affine root systems,
- build Bershtein-Gelfand-Gelfand resolution for representations of finite and affine Lie algebras and prove the Weyl-Kac formula for their characters.

Affine Lie algebras also play an important role in physics. It turns out that a two-dimensional sigma-model on a group manifold can be made conformal by adding a special topological term to the action. The symmetry of the resulting theory (Wess-Zumino-Novikov-Witten model) is actually much larger than the standard Virasoro algebra of  $2d$  CFTs — it is an affine Lie algebra. WZNW models will be our second focus in this course.

Affine Lie algebra symmetry of WZNW models has several important consequences:

- The correlators of the models obey first-order Knizhnik-Zamolodchikov differential equations, which in many cases can be solved explicitly.
- The partition function of the model on a torus can be expressed as a bilinear combination of a finite number of affine Lie algebra characters.
- Modular invariance of the partition function implies modular properties of characters.

If time permits we will also mention the connections of WZNW models with Chern-Simons theory, quantum groups and knot invariants.

## References

- [1] P. Di Francesco, P. Mathieu, D. Sénéchal, Conformal Field Theory, Springer-Verlag (1997)
- [2] V. Kac, Infinite dimensional Lie algebras (3 ed.), Cambridge University Press (1990)
- [3] J. Fuchs, Affine Lie Algebras and Quantum Groups, Cambridge University Press (1992)