# SMOOTH 4-MANIFOLDS

- **Lecturer:** Rafael Torres
- **Course Type:** PhD Course
- **Academic Year:** 2015-2016
- **Term:** Spring 2016
- **Duration** 40 hours
- **Schedule** Tuesdays and Thursdays, 11 - 12:30 (Tentative)
- **Location** SISSA, Room TBA
- **Contact Info:** rtorres@sissa.it. Office 711, SISSA.

## 1. Syllabus

The following is an outline of the material we aim at covering through out the course. Changes might (and most likely will) occur as the course progresses.

1.1. **Introduction and Perspectives.** Statement of goals of the course. Comparison of four-dimensional manifolds to their counterparts in other dimensions. Classification of surfaces. Smale's $h$-cobordism theorem and its corollary, the generalized Poincaré conjecture. Fundamental dimensional difference: TOP $\neq DIFF$ and intersections of submanifolds.

1.2. **Invariants to determine a homeomorphism class.** Intersection forms over $\mathbb{Z}$, and their invariants: Euler characteristic, signature, second Stiefel-Whitney class. Understanding 4-manifolds thourgh their submanifolds: counting intersections. Freedman's topological classification of closed 4-manifolds with fundamental group $\pi_1 = \{1\}$. Donaldson's results on definite forms that can be realized by smooth 4-manifolds [4, Chapter 1], [3, Chapter II].

1.3. **Examples of smooth 4-manifolds and computations.** Examples of 4-manifolds and their intersection forms: $\mathbb{CP}^2$ and its blow ups, Hirzebruch surfaces, and complex surfaces in general. Two ways of looking at the K3 surface [4, Chapter 1]. Exotic $\mathbb{R}^4$'s [3, Chapter XIV].

1.4. **Constructions of 4-manifolds.** Gluck twists, torus and Knot surgeries.

1.5. **Using submanifolds to understand 4-manifolds.** Surfaces in 4-manifolds [4, Sections 2.1, 2.2, 2.3]. Surfaces of minimal genus, the adjunction inequality.

1.6. **Geometric structures I: Obstructions.** We study spin, almost-complex, complex, and symplectic structures. Explicit examples and restrictions on the topology of the underlying manifolds will be discussed.

1.7. **Overview of results in Seiberg-Witten theory.** Obstructions to existence of Riemannian metrics of positive scalar curvature and to the existence of a symplectic structure.

1.8. **Geometric structures II: Symplectic 4-manifolds.** Constructions of symplectic 4-manifolds[4]. Structure results: Lefschetz pencils, Lefschetz fibrations,and their topology, Donaldson's and Gompf's results [4, Section 8.1, 8.2].

1.9. **Geometric structures III: Complex structures.** Complex surfaces. Kodaira classification [2, 4]. Elliptic fibrations. Classification of their fibers. Log transforms. Exotic examples, and diffeomorphism types [2, 4].

1.10. **Geometric structures IV: Riemannian geometry in dimension four.** Splitting of the curvature tensor of a Riemannian 4-manifold and its implications. Einstein metrics. Smooth structures and their geometry.

## References

[1] S. Akbulut, *4-manifolds* Lecture notes (2015)
available at http://users.math.msu.edu/users/Akbulut/papers/akbulut.lec.pdf
[2] W.P. Barth, K. Hulek, C.A.M. Peters and A. van de Ven, *Compact Complex Surfaces*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3), vol. 4, Springer, Berlin, Second Enlarged Edition, 2004.
[3] R. Kirby, *The topology of 4-manifolds* LNM 1374.
[4] R. E. Gompf and A. I. Stipsicz, *4-Manifolds and Kirby Calculus*. Graduate Studies in Mathematics, 20. AMS, Providence, RI, 1999.