Long-time asymptotics for the Camassa-Holm equation
10 lectures, 20 hours
Alexander Minakov, Fall 2015

The course is based on the papers [1], [2]. The aim is to derive the asymptotics for the solution $u(x, t)$ of the Cauchy problem for the Camassa-Holm equation

$$u_t + 2u_x - u_{txx} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad x \in \mathbb{R}, t \geq 0, \quad (1)$$

with vanishing initial data

$$u(x, t = 0) = u_0(x). \quad (2)$$

The exposition inculdes:
– Lax pair: overdetermined system of linear ordinary differential equations;
– Sturm -Liouville equation, Liouville transform, Jost solutions;
– transmission and reflection coefficients;
– Riemann-Hilbert problem;
– asymptotic analysis of solutions of oscillatory Riemann-Hilbert problems: nonlinear steepest descent method, explicitly solvable in terms of special functions model Riemann-Hilbert problems.

References

[1] Boutet de Monvel, Anne; Shepelsky, Dmitry. *Riemann-Hilbert problem in the inverse scattering for the Camassa-Holm equation on the line.* Probability, geometry and integrable systems, Math. Sci. Res. Inst. Publ., Cambridge Univ. Press, Cambridge, (2007), no. 55, 2007, 53-75.

[2] Boutet de Monvel, Anne; Kostenko, Aleksey; Shepelsky, Dmitry; Teschl, Gerald *Long-time asymptotics for the Camassa-Holm equation.* SIAM J. Math. Anal., (2009), no. 4/41, 1559–1588.