

Introduction to Linear Differential Equations

in the Complex Domain – 10 lectures

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- Motivations and basic facts. Existence theorems, uniqueness theorems. Real, complex and analytic case. Equations of order n and differential systems of order 1 and dimension n .
- Linear differential systems. Analyticity properties of solutions, analytic continuation.
- Singularities of differential equations. Singularities of linear differential equations: monodromy group, monodromy representation.
- Linear systems with singularities of the first kind. Local behavior of solutions. Reduction to normal form (formal theory and analytic theory) and construction of local solutions in normal form.
- Miscellaneous remarks on equations of order $n \geq 2$. Hyper-geometric equation.
- Introduction to asymptotic expansions.
- Linear systems with singularities of the second kind. Reduction to normal form (formal theory and analytic theory, non-ramified and ramified cases).
- Stokes phenomenon. Stokes matrices. Some examples, such as Bessel equation and Airy equation.
- Global description. Monodromy data.
- If time allows, applications to non linear equations will be sketched (method of isomonodromy deformations).

Prerequisites: Complex analysis, theory of analytic functions in one complex variable (see Reference 4. below). .

Main References

1. W. Wasow: *Asymptotic Expansions for Ordinary Differential Equations*.

2. E.A. Coddington, N. Levinson: *Theory of Ordinary Differential Equations*.
3. E.L. Ince: *Ordinary Differential Equations*
4. V.I. Smirnov: *A course of higher mathematics. Vol. 3. Part 2: complex variables, special functions*
5. K. Iwasaki, H. Kimura, S. Shimomura, M. Yoshida: *From Gauss to Painlevé*.
6. W. Balser, W.B. Jurkat, D.A. Lutz: "*Birkhoff Invariants and Stokes' Multipliers for Meromorphic Linear Differential Equations*". *Journal of Mathematical Analysis and Applications* **71**, 48-94 (1979).
7. A. Fokas, A.R. Its, A.A. Kapaev, V.Y. Novokshenov: *Painlevé Transcendents, The Riemann-Hilbert approach*.
8. Y. Sibuya: *Linear Differential Equations in Complex Domain; Problems of Analytic Continuation*. *Translations of Mathematical Monographs* 82, AMS.