

Random Matrices: Tentative Course Outline

Contents

1 Introduction: Origin in Nuclear physics. Connections with number theory and Enumerative Geometry, Integrable Systems	3
2 Wigner and Wishart matrices	3
2.1 Statement of spectral problem	3
2.2 The moment method; combinatorial approach and Dyck paths. Catalan numbers	3
2.3 Convergence of the empirical measure in moments and in probability. Reminder of Markov and Chebycheff inequalities	3
3 More combinatorics; Gaussian integrals and enumeration of maps	3
4 Invariant Ensembles	3
4.1 Unitary, Orthogonal and Symplectic ensembles; Weyl integration formula and reduction to eigenvalues	3
4.2 Unitary Ensembles	3
4.2.1 Dyson theorem ("integrating out")	3
4.2.2 Orthogonal polynomials	3
5 GUE again	3
5.1 Hermite polynomials; integral formulas	3
5.2 Kernel and asymptotics; steepest descent method for contour integrals in the complex plane	3
5.3 Sine kernel in the bulk	3
5.4 Airy kernel at the edge	3
5.5 Riemann–Hilbert problems; generalities	3
5.6 Gap probabilities and Tracy-Widom distribution	3
5.6.1 Painlevé II equation and RHP	3
6 Determinantal Random Point Fields (DRPF)	3
6.1 Generalities and main properties	3
6.2 Fredholm determinants; integrable kernels and IIKS	3
6.3 Loop Equations: topological recursion of Eynard-Orantin	3
7 Orthogonal polynomials	3
7.1 General properties; Christoffel–Darboux identity	3
7.2 Riemann–Hilbert formulation	3
7.3 Connection with integrable hierarchies: Kadomtsev-Petviashvili and Toda	3
8 Nonabelian Steepest Descent Method of Deift-Zhou	3
8.1 Elements of potential theory	3
8.2 The small–norm theorem for Riemann-Hilbert problems	3
8.3 Sine-kernel universality in the bulk	3

1 Introduction: Origin in Nuclear physics. Connections with number theory and Enumerative Geometry, Integrable Systems

2 Wigner and Wishart matrices

2.1 Statement of spectral problem

2.2 The moment method; combinatorial approach and Dyck paths. Catalan numbers

2.3 Convergence of the empirical measure in moments and in probability. Reminder of Markov and Chebycheff inequalities

3 More combinatorics; Gaussian integrals and enumeration of maps

4 Invariant Ensembles

4.1 Unitary, Orthogonal and Symplectic ensembles; Weyl integration formula and reduction to eigenvalues

4.2 Unitary Ensembles

4.2.1 Dyson theorem ("integrating out")

4.2.2 Orthogonal polynomials

5 GUE again

5.1 Hermite polynomials; integral formulas

5.2 Kernel and asymptotics; steepest descent method for contour integrals in the complex plane

5.3 Sine kernel in the bulk

5.4 Airy kernel at the edge

5.5 Riemann–Hilbert problems; generalities

5.6 Gap probabilities and Tracy-Widom distribution

5.6.1 Painlevé II equation and RHP

6 Determinantal Random Point Fields (DRPF)

6.1 Generalities and main properties

6.2 Fredholm determinants; integrable kernels and IIKS

6.3 Loop Equations: topological recursion of Eynard-Orantin

7 Orthogonal polynomials

7.1 General properties; Christoffel–Darboux identity

7.2 Riemann–Hilbert formulation