ALGEBRAIC GEOMETRY, NEWTON POLYHEDRA AND NEWTON–OKOUNKOV BODIES

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Newton polyhedra connect algebraic geometry and the theory of singularities to the geometry of convex polyhedra. This connection is useful in both directions.

On the one hand, explicit answers are given to problems of algebra and the theory of singularities in terms of the geometry of polyhedra.

On the other hand, algebraic theorems of general character give significant information about the geometry of polyhedra.

To arbitrary finite-dimensional space of rational functions L on any (not necessarily complete) fixed irreducible n-dimensional algebraic variety X one associates its Newton –Okounkov body $\Delta(L) \subset \mathbb{R}^n$. The construction of the body $\Delta(L)$ is nonunique and contains arbitrary functional parameters. It is based on a new geometric theory of semigroups of integral points.

The theory of Newton–Okounkov bodies relates algebra and geometry outside the framework of toric geometry. This relationship is useful in many directions. For algebraic geometry it provides elementary proofs of intersection-theoretic analogues of the geometric Alexandrov-Fenchel inequalities. In abstract algebra it allows one to introduce a broad class of graded algebras whose Hilbert functions are not necessarily polynomial at large values of the argument but have polynomial asymptotics. The local version of the theory provides new inequalities (similar to the reverse AlexandrovFenchel inequalities) for the multiplicities of primary ideals in a local ring. In geometry it provides a transparent proof of the known Alexandrov-Fenchel inequality and for its new analog for coconvex bodies.

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